Answers

Exercise 1A. Page 18.

1.	36	**2.**	8	**3.**	16	**4.**	1	**5.**	1
6.	2	**7.**	1	**8.**	2	**9.**	1	**10.**	8
11.	1	**12.**	-1	**13.**	-32	**14.**	1	**15.**	-1
16.	4	**17.**	5	**18.**	2	**19.**	3	**20.**	9
21.	$\frac{1}{9}$	**22.**	$\frac{1}{4}$	**23.**	$\frac{1}{8}$	**24.**	$\frac{3}{4}$	**25.**	$\frac{1}{6}$
26.	7	**27.**	5	**28.**	$\frac{1}{5}$	**29.**	1	**30.**	1
31.	1	**32.**	-2	**33.**	125	**34.**	$\frac{4}{3}$	**35.**	$\frac{2}{3}$
36.	27	**37.**	$\frac{1}{27}$	**38.**	$\frac{1}{16}$	**39.**	$\frac{3}{2}$	**40.**	25
41.	25	**42.**	$\frac{1}{16}$	**43.**	$\frac{1}{512}$	**44.**	2^{16}	**45.**	2^{13}
46.	2^6	**47.**	2^{14}	**48.**	2^8	**49.**	2^0	**50.**	2^{-1}
51.	2^{-3}	**52.**	2^{-3}	**53.**	3^3	**54.**	3^4	**55.**	3^0
56.	3^{-1}	**57.**	$3^{0.5}$	**58.**	$3^{0.25}$	**59.**	3^{-3}	**60.**	$3^{-0.5}$
61.	3^6	**62.**	10^2	**63.**	10^{-1}	**64.**	10^{-1}	**65.**	10^{-2}
66.	10^{-2}	**67.**	10^0	**68.**	10^6	**69.**	10^6	**70.**	10^6
71.	10^9	**72.**	10^{-3}	**73.**	$10^{0.5}$	**74.**	10^3	**75.**	$10^{-0.5}$
76.	$10^{\frac{3}{2}}$	**77.**	5	**78.**	4	**79.**	3	**80.**	4
81.	7	**82.**	5	**83.**	8	**84.**	8	**85.**	10
86.	8	**87.**	9	**88.**	8	**89.**	10	**90.**	8
91.	7	**92.**	3	**93.**	2	**94.**	2	**95.**	15
96.	3	**97.**	4	**98.**	a^7	**99.**	$\frac{1}{a^3}$	**100.**	$\frac{1}{b^5}$
101.	b^6	**102.**	$\frac{1}{b^6}$	**103.**	a^2	**104.**	$\frac{1}{a^6}$	**105.**	a^2
106.	b^{10}	**107.**	b^{18}	**108.**	$-4a^5$	**109.**	$16a^5$	**110.**	$\frac{1}{a^2}$
111.	$\frac{a^9}{b^3}$	**112.**	$6a^3$	**113.**	$\frac{1}{a^3}$	**114.**	a^2	**115.**	$2a$
116.	$8a^5$	**117.**	$32a^5$	**118.**	$4a^2b^4$	**119.**	$\frac{10y^2}{a^5}$	**120.**	$\frac{3}{2a^4b^3}$
121.	$\frac{y}{3x}$	**122.**	$-9a^8b$	**123.**	a^2b^5	**124.**	$\frac{3a^4}{2b^2}$	**125.**	$\frac{1}{a^5}$
126.	$\frac{y^3}{x^4}$	**127.**	$\frac{b^4}{a^8}$	**128.**	$a^4 + a$	**129.**	$a^2 + a^4$	**130.**	$2a + 3$
131.	$\frac{4}{5}$	**132.**	$7(2^n)$	**133.**	9				

Exercise 1B. Page 23.

1.	$x = 3$	**2.**	$x = 5$	**3.**	$x = 7$	**4.**	$x = -3$	**5.**	$x = -5$
6.	$x = -7$	**7.**	$a = 0{\cdot}5$	**8.**	$a = -0{\cdot}5$	**9.**	$y = -2$	**10.**	$c = 3$
11.	$d = 3$	**12.**	$x = 3$	**13.**	$x = 3$	**14.**	$x = 6$	**15.**	$x = 2$
16.	$x = 1$	**17.**	$x = 0{\cdot}75$	**18.**	$x = -1$	**19.**	$x = 3$	**20.**	$x = 2$
21.	$x = 5$	**22.**	$k = 0{\cdot}75$	**23.**	$p = -0{\cdot}25$	**24.**	$x = 1{\cdot}5$	**25.**	$x = 1$
26.	$h = 6$	**27.**	$x = 0{\cdot}5$	**28.**	$x = 1$	**29.**	$x = -0{\cdot}25$	**30.**	$n = 2$
31.	$a = \pm 4$	**32.**	$p = \pm 10$	**33.**	$x = 2$	**34.**	$x = 4$	**35.**	$x = 16$
36.	$x = 64$	**37.**	$h = 0{\cdot}25$	**38.**	$y = 0{\cdot}5$	**39.**	$p = 3$	**40.**	$x = 10000$
41.	$x = \pm 5$	**42.**	$x = \pm\frac{2}{3}$	**43.**	$x = \pm 3$	**44.**	$x = 1000$	**45.**	$p = \pm 4$
46.	$x = 4$	**47.**	$x = 81$	**48.**	$x = \frac{1}{16}$	**49.**	$w = 4$	**50.**	$x = 2$
51.	$h = 24$	**52.**	$x = -1$ or 7	**53.**	$w = \frac{9}{4}$ i.e. $2{\cdot}25$	**54.**	$z = \frac{27}{8}$ i.e. $3{\cdot}375$		

55. (a) The third solution is $x = 0$. (b) $x = 0,\ x = -4,\ x = 4$
(c) (i) $x = 0,\ x = 4$ (ii) $x = 0,\ x = -5,\ x = 5$ (iii) $x = 0,\ x = 25$ (iv) $x = 0$

Exercise 1C. Page 26.

1.	$x = 4{\cdot}5$	**2.**	$x = 3{\cdot}2$	**3.**	$x = 2{\cdot}4$	**4.**	$x = 2{\cdot}8$
5.	$x = 6{\cdot}7$	**6.**	$x = 2{\cdot}2$	**7.**	$x = 2{\cdot}46$	**8.**	$x = 2{\cdot}55$
9.	$x = 3{\cdot}32$	**10.**	$x = 6{\cdot}26$	**11.**	$x = 3{\cdot}86$	**12.**	$x = 3{\cdot}53$
13.	$x = 8{\cdot}43$	**14.**	$x = -1{\cdot}51$	**15.**	$x = 4{\cdot}35$	**16.**	$x = 3{\cdot}34$
17.	$x = 2{\cdot}74$	**18.**	$x = -6{\cdot}99,\ x = 3{\cdot}38$				

Miscellaneous Exercise One. Page 27.

1. Completed table not given here. $y = -x^3 + 2x^2 - x + 7$

2. (a) Australia has an area of approximately 7 682 000 km^2.
(b) Light travels at a speed of 300 000 000 m/sec.
(c) A golf ball has a mass of approximately 0·045 kg.
(d) The earth is approximately 150 000 000 km from the sun.
(e) Gamma waves have a wave length less than 0·000 000 000 01 metres.
(f) The earth orbits the sun at a speed of approximately 107 000 km/hr.
(g) In 1961 the first man in space, Yuri Gagarin, flew his spacecraft at a speed of 27 400 km/hr, i.e. approximately 7 600 m/sec.

3. (a) At the beginning of this century China had a population of approximately $1{\cdot}27 \times 10^9$ and India had a population of approximately $1{\cdot}03 \times 10^9$.
(b) The egg cell, or ovum, with a radius of approximately 5×10^{-5} metres, i.e. 5×10^{-2} mm, is the largest single human cell.
(c) It is thought that approximately $1{\cdot}1 \times 10^6$ people die each year of Malaria.
(d) Some adult wasps of a particular species could weigh just 5×10^{-3} grams.
(e) Concorde, the first supersonic passenger airliner, had a cruising speed of $2{\cdot}16 \times 10^3$ km/hr.

4. (a) 12 000 000 (b) 46 800 (c) 305 000 000
(d) 0·01 (e) 0·206 (f) 0·006

5. (a) 5^2 (b) 5^3 (c) $5^{0.5}$ (d) 5^{-1}
(e) 5^{-2} (f) $5^{-0.5}$ (g) 5^7 (h) 5^7
(i) 5^6 (j) 5^{12} (k) 5^6 (l) 5^0
(m) 5^5 (n) 5^2 (o) 5^5

6. (a) $x = 5$ (b) $x = 4$ (c) $x = -2$ (d) $x = \frac{5}{2}$

(e) $x = \frac{5}{3}$ (f) $x = \frac{1}{3}$ (g) $x = -\frac{1}{3}$ (h) $x = -3$

(i) $x = -\frac{2}{5}$ (j) $x = \frac{1}{4}$ (k) $x = -\frac{3}{2}$ (l) $x = 8$

7. (a) $y = \pm\frac{1}{3}$ (b) $p = \frac{4}{9}$ (c) $x = \frac{3}{2}$ (d) $x = 8$

(e) $x = \pm 5$ (f) $t = \pm\frac{1}{5}$ (g) $t = \frac{27}{2}$ (h) $x = 9$

(i) $x = 0, x = 1$ (j) $x = 0, x = \pm 1$

8. (a) $x = 1,\ x = 3$ (b) $x = 0,\ x = 2$ (c) $x = -1,\ x = 0$ (d) $x = 1,\ x = 3$

Exercise 2A. Page 33.

1. Rule: $y = 3^x$

x	0	1	2	3	4	5
y	1	3	9	27	81	243

2. Rule: $y = 7^x$

x	0	1	2	3	4	5
y	1	7	49	343	2401	16807

3. Rule: $y = 1{\cdot}5 \times 2^x$

x	0	1	2	3	4	5
y	1·5	3	6	12	24	48

4. Rule: $y = 1{\cdot}75 \times 8^x$

x	0	1	2	3	4	5
y	1·75	14	112	896	7168	57344

5. Rule: $y = 2^{x+1}$

x	0	1	2	3	4	5
y	2	4	8	16	32	64

6. Rule: $y = 2{\cdot}5 \times 4^{x+1}$

x	1	2	3	4	5	6
y	40	160	640	2560	10240	40960

7. (a) Quadratic (b) $y = x^2 + 1$ **8.** (a) Exponential (b) $y = 4^x$

9. (a) Linear (b) $y = 2x + 3$ **10.** (a) Quadratic (b) $y = 2x^2$

11. (a) Exponential (b) $y = 1{\cdot}5(8)^x$ **12.** (a) Exponential (b) $y = 5^x$

13. (a) Quadratic (b) $y = x^2 + x$ **14.** (a) Exponential (b) $y = 6^x$

15. (a) Exponential (b) $y = 3(2)^x$ **16.** (a) Reciprocal (b) $y = \frac{60}{x}$

17. (a) Cubic (b) $y = x^3 + 1$ **18.** (a) Linear (b) $y = 20 - 3x$

19. (a) (0, 1) (b) Discuss your answer with your teacher.

20. (b) and **21** and **22.** Discuss your answer with your teacher.

23. (a) $y = 2^x$ (b) $y = 3^x$ **24.** (a) 12 (b) 28 (c) 40

25. (a) $y = 3^{x-2}$ (b) $y = 2^x + 2$ (c) $y = 2^{x-2}$

(d) $y = 3^x - 2$ (e) $y = 3^{x+1} + 2$ (f) $y = 2^{x-2} - 2$

Exercise 2B. Page 39.

3. Approximately 61 million, assuming annual growth rate for the given years continues.

4. Approximately 6000

5. (a) A = 80, k = 1·08 (b) 8% (c) 1200

6. Approximately 1 200

7. (a) 68 (b) 29 (c) 0·84, 115 (d) 115 (e) 14

8. (a) $k \approx 80$, $a \approx 0{\cdot}92$ (b) Approximately 27 (c) Approximately 2021

9. (a) 10000 of A and 1000 of B (b) 4200 of A and 1300 of B (c) 6·2

10. (a) $k \approx 850$, a is between 0·9 and 0·91 (b) Approximately 14 weeks

Miscellaneous Exercise Two. Page 43.

1. (a) II (b) IV (c) III (d) I (e) III (f) IV (g) III (h) I

2. (a) $x = \pm 7$ (b) $x = \pm 10$ (c) $x = 10$ (d) $x = 2$ (e) $x = 4$ (f) $x = 0$
(g) $x = 3$ (h) $x = -1$ (i) $x = -2$ (j) $x = -3$ (k) $x = -1$ (l) $x = -2$
(m) $x = -3$ (n) $x = 0, \pm 5$ (o) $x = -0{\cdot}125$ (p) $x = 0{\cdot}5$ (q) $x = 0{\cdot}25$ (r) $x = \pm 3$

3. (a) 12 000 (b) 12 610 000 (c) 0·000 26 (d) 6 (e) 12 630

4. (a) $y = 2^{x+3}$, $y = 8 \times 2^x$ (b) $y = 3^x - 2$

5. Check reasonableness of answers by evaluating $5^{1{\cdot}6}$, $5^{2{\cdot}4}$ and $5^{2{\cdot}5}$ on a calculator.

6. −2 or 0 **7.** 0 or 2

8. (a) $k \approx 18{\cdot}9$, $a \approx 0{\cdot}93$ (b) Approx 6.22 am (Remember graph shows number of *half* hours.)

Exercise 3A. Page 48.

1.	18	**2.**	26	**3.**	44	**4.**	38	**5.**	42	**6.**	36	**7.**	72	**8.**	44
9.	42	**10.**	46	**11.**	324	**12.**	2744	**13.**	8	**14.**	20	**15.**	28	**16.**	26
17.	29	**18.**	21	**19.**	27	**20.**	9	**21.**	162	**22.**	18	**23.**	26	**24.**	4374
25.	27	**26.**	216	**27.**	343	**28.**	91	**29.**	4	**30.**	7	**31.**	49	**32.**	94

Exercise 3B. Page 54.

1. $T_1 = 6$, $T_{n+1} = T_n + 4$.

2. $T_1 = 28$, $T_{n+1} = T_n - 2$.

3. $T_1 = 5$, $T_{n+1} = T_n + 10$.

4. $T_1 = 7{\cdot}5$, $T_{n+1} = T_n + 2{\cdot}5$.

5. $T_1 = 100$, $T_{n+1} = T_n - 11$.

6. $T_1 = 6$, $T_n = 2T_{n-1}$.

7. $T_1 = 0{\cdot}375$, $T_n = 4T_{n-1}$.

8. $T_1 = 384$, $T_n = 0{\cdot}25T_{n-1}$.

9. $T_1 = 50$, $T_n = 3T_{n-1}$.

10. $T_1 = 1000$, $T_n = 1{\cdot}1T_{n-1}$.

11. Neither arithmetic nor geometric.

12. Geometric.

13. Arithmetic.

14. Arithmetic.

15. Neither arithmetic nor geometric.

16. Geometric.

17. Geometric.

18. Arithmetic.

19. Neither arithmetic nor geometric.

20. Neither arithmetic nor geometric.

21. Arithmetic.

22. Geometric.

23. $T_1 = 8$, $T_2 = 11$, $T_3 = 14$, $T_4 = 17$, $T_{n+1} = T_n + 3$.

24. $T_1 = 100$, $T_2 = 97$, $T_3 = 94$, $T_4 = 91$, $T_{n+1} = T_n - 3$.

25. $T_1 = 11$, $T_2 = 22$, $T_3 = 44$, $T_4 = 88$, $T_{n+1} = 2T_n$.

26. $T_1 = 2048$, $T_2 = 1024$, $T_3 = 512$, $T_4 = 256$, $T_{n+1} = 0{\cdot}5T_n$.

27. (b) $N_{n+1} = N_n + 800$

28. (a) The sequence is a geometric progression.
(b) The next three terms after the first are 550, 605 and 665·5.

29. (a) The sequence is a geometric progression.
(b) The next three terms after the first are1250, 1562·5 and 1953·125.

30. (a) The sequence is a geometric progression.
(b) The next three terms after the first are 21600, 19440 and 17496.

31. (a) $T_1 = 3$, $T_{n+1} = T_n + 5$. (b) The sequence is arithmetic.

32. (a) $T_1 = 1{\cdot}5$, $T_{n+1} = T_n \times 2$, (b) The sequence is geometric.

33. (a) $T_1 = 4$, $T_2 = 9$, $T_3 = 16$, $T_4 = 25$, $T_5 = 36$. (b) Neither arithmetic nor geometric.

34. (a) After 1 year, 2 years, 3 years and 4 years the account is worth \$1 296, \$1 392, \$1 488 and \$1 584 respectively.
(b) The amounts are in arithmetic progression. (c) $T_1 = \$1200$, $T_{n+1} = T_n + \$96$.

35. $T_1 = 4$, $T_{n+1} = T_n + 1$.

36. $T_1 = \$45000$, $T_{n+1} = T_n + \$1500$. The terms of the sequence progress arithmetically.

37. \$68 000 in 2014, \$71 400 in 2015, \$74 970 in 2016, \$78 718·50 in 2017.
$T_1 = \$68\,000$, $T_{n+1} = 1{\cdot}05T_n$.

38. $T_1 = \$1500$, $T_{n+1} = 1{\cdot}08T_n$.

39. $T_1 = \$36\,000$, $T_{n+1} = 0{\cdot}85T_n$.

Exercise 3C. Page 64.

1. $T_{100} = 506$ **2.** $T_{100} = 289$ **3.** $T_{100} = 815$

4. $T_{100} = -120$ **5.** $T_{25} = 5 \times 2^{24}$ **6.** $T_{25} = 1{\cdot}5 \times 4^{24}$

7. $T_{25} = 8 \times 3^{24}$ **8.** $T_{25} = 11 \times 2^{24}$ **9.** $T_{28} = 223$

10. $T_{20} = 3\,495\,265$ **11.** $T_{19} = 774\,840\,977$ **12.** $T_{45} = 6$

13. $T_1 = 48$, $T_{n+1} = T_n + 3$. Julie successfully completes 90 items on the 15th day.

14. Substituting y for T_n and x for n the rule $T_n = a + (n - 1)d$ becomes $y = dx + (a - d)$.
This is the equation of a straight line with gradient d, cutting the y-axis at $(0, a - d)$.

15. Substituting y for T_n and x for n the rule $T_n = ar^{n-1}$ becomes $y = ar^{x-1}$, i.e. $y = \left(\frac{a}{r}\right) r^x$.
An exponential function cutting the y-axis at $\left(0, \frac{a}{r}\right)$.

16. As $n \to \infty$ the "nd" term in the expression $a + (n - 1)d$ will dominate.
Thus as $n \to \infty$, T_n will be increasingly large and positive if $d > 0$
and increasingly large and negative if $d < 0$.

17. As $n \to \infty$ the n in the expression ar^{n-1} will dominate.
Thus as $n \to \infty$, if $r > 1$, T_n will become increasingly large, either positively or negatively dependent on the sign of the constant a.
if $r < -1$, T_n will become increasingly large, alternating between large negative and large positive.
if $-1 < r < 1$, T_n will become smaller and smaller, maintaining the sign of the constant a if r is positive and alternating between small positive and small negative if r is negative.
I.e., if $-1 \leq r \leq 1$, as $n \to \infty$, $T_n \to 0$.

18. The first four terms are 8, 11, 14, 17. The 50th term is 155. The 100th term is 305.

19. The first four terms are 100, 97, 94, 91. The 50th term is −47. The 100th term is −197.

20. 11, 22, 44, 88, 180 224, 184 549 376

21. 2048, 1024, 512, 256, 0·0625

22. (a) $T_n = 9 + (n - 1) \times 6$, i.e. $T_n = 6n + 3$ (b) $T_n = 7 + (n - 1) \times 1{\cdot}5$, i.e. $T_n = 1{\cdot}5n + 5{\cdot}5$

23. (a) $3 \times 2^{n-1}$ (b) $100 \times 1{\cdot}1^{n-1}$

24. (a) 856 (b) 3495 (c) The 142 858th term

25. (a) 126 953·125 (b) The 14th term

26. (a) 844 700 (b) The 60th term

27. 1, 8, 27, 64, neither

28. (a) 64 (b) 7

29. (a) 1 850 (b) 2 000

30. (a) 7 971 615 (b) 5

31. (a) −1 835 008 (b) 7

32. The amount in the account at the end of ten years is $8635·70

33. Just after the end of the 22nd year, i.e. early in the 23rd year.

34. $T_{n+1} = 1{\cdot}08 \times T_n + \$200$, $T_{10} = \$11533{\cdot}01$

35. $T_{n+1} = 1{\cdot}08 \times T_n - \$200$, $T_{10} = \$5738{\cdot}39$

Miscellaneous Exercise Three. Page 67.

1. (a) Quadratic (b) Exponential (c) Linear
(d) Quadratic (e) Reciprocal (f) Linear
(g) Linear (h) Quadratic (i) Quadratic
(j) Reciprocal (k) Linear (l) Exponential

2. (a) $x \approx 2{\cdot}3$ (b) $x \approx 2{\cdot}6$ (c) $x \approx 1{\cdot}4$

3. (a) 3 (b) −3 (c) −1 (d) 0·5 (e) 0 (f) 1·5
(g) −6 (h) 1·5

4. (a) $T_n = 4 \times 1{\cdot}5^{n-1}$ (b) $T_n = \frac{8}{3} \times 1{\cdot}5^n$

5. 243

6. (a) $x = 3$ (b) $x = -3$ (c) $x = -1$ (d) $x = 8$ (e) $x = \pm 8$ (f) $x = 11$

7. (a) 4 (b) 64 (c) 9 (d) 0·2 (e) 2

8. (a) –4, (b) –4, 4, 4 (c) Neither

9. (a) –2, (b) –4, 8, 16 (c) Neither

10. (a) 48 – 4k (b) 95·5

11. a^7 **12.** $12x^3y^4$ **13.** $\frac{3a^2}{2b^2}$

14. $72a^8b^3$ **15.** $\frac{9}{8a^4b^3}$ **16.** $\frac{48b}{a}$

17. $\frac{2a^5}{b^5}$ **18.** $k^4 + 1$ **19.** $p^3 - p^6$

20. 125 **21.** 25 **22.** $\frac{8}{3}$

Exercise 4A. Page 73.

1. (a) 68 (b) 100 (c) 138 **2.** (a) 53 (b) 123 (c) 28 **3.** (a) –9 (b) 0 (c) 9

4. $T_1 = 6$, $T_2 = 11$, $T_3 = 16$, $T_4 = 21$. $S_1 = 6$, $S_2 = 17$, $S_3 = 33$, $S_4 = 54$.

5. $T_1 = 11$, $T_2 = 14$, $T_3 = 17$, $T_4 = 20$. $S_1 = 11$, $S_2 = 25$, $S_3 = 42$, $S_4 = 62$.

6. $T_1 = 22$, $T_2 = 19$, $T_3 = 16$, $T_4 = 13$. $S_1 = 22$, $S_2 = 41$, $S_3 = 57$, $S_4 = 70$.

7. $T_1 = 25$, $T_2 = 32$, $T_3 = 39$, $T_4 = 46$. $T_5 = 53$. Yes

8. $T_1 = 1$, $T_2 = 4$, $T_3 = 9$, $T_4 = 16$. $T_5 = 25$. No

9. (a) 48 (b) 8 780 **10.** (a) 174 (b) 60 **11.** 5 050

12. (b) 2 088 **13.** (b) 14 309 **14.** 78 km, 1 470 km

15. 285 **16.** $31 500 **17.** A: $762 500 B: $734 000

18. $6 840

Exercise 4B. Page 77.

1. $T_1 = 6$, $T_2 = 18$, $T_3 = 54$, $T_4 = 162$. $S_1 = 6$, $S_2 = 24$, $S_3 = 78$, $S_4 = 240$.

2. $T_1 = 16$, $T_2 = 24$, $T_3 = 36$, $T_4 = 54$. $S_1 = 16$, $S_2 = 40$, $S_3 = 76$, $S_4 = 130$.

3. $T_1 = 1$, $T_2 = 1$, $T_3 = 2$, $T_4 = 3$. $T_5 = 5$. No

4. $T_1 = 8$, $T_2 = 24$, $T_3 = 72$, $T_4 = 216$. $T_5 = 648$. Yes

5. 32 767 **6.** 40 940 **7.** 650 871

8. 104 139·36 **9.** 3 071·25, 12 287·25, 49 151·25 **10.** 20

11. 25 **12.** 393 216, 524 287·5 **13.** $1 015 000

14. (a) Approximately 5 500 tonnes (b) Approximately 6 050 tonnes
(c) Approximately 6 655 tonnes (d) Approximately 107 000 tonnes

15. (a) Approximately $69 000 (b) Approximately $79 350
(c) Approximately $211 000 (d) Approximately $1 218 000

16. Entries in last line of table are:

1/1/18 $\$1200 \times 1{\cdot}1^4$ $\$1200 \times 1{\cdot}1^3$ $\$1200 \times 1{\cdot}1^2$ $\$1200 \times 1{\cdot}1$ $1200 $7326·12

Immediately following the deposit of $1200 on 1/1/29 there will be $43 140 in the account, to the nearest dollar.

17. $14 784

18. (a) Approximately 2 653 (b) Approximately 3 299 (c) Approximately 3 299
(d) Approximately 43 200 (e) Approximately 125 700

19. (a) 17 years (b) Approximately 79 500 tonnes

20. (a) First term $P, common ratio 1·095, number of terms 21. (b) 829·7

Exercise 4C. Page 83.

1. GP A: (a) 0·4 (b) S_∞ exists and equals 40.
GP B: (a) 1·5 (b) S_∞ does not exist.
GP C: (a) 0·3 (b) S_∞ exists and equals 50.

2. (a) S_∞ exists and equals 200. (b) S_∞ exists and equals 400.
(c) S_∞ does not exist. (d) S_∞ exists and equals 450.
(e) S_∞ does not exist. (f) S_∞ exists and equals 50.
(g) S_∞ exists and equals 0·9. (h) S_∞ exists and equals 2048.

3. 0·6 **4.** 66

5. Table not shown here. (a) 25 mg (b) 10 mg

6. 250.
The idea that the athlete's performance might diminish according to some geometrical pattern is not unreasonable as he would tire as time went on. Hence the situation could feasibly be modelled by a geometrical sequence but we would be surprised if the numbers exactly fitted the model.
However, if the geometrical sequence were continued, by the 10th minute the athlete is completing the exercise approximately 7 times and by the 15th minute approximately 2 times so it could be argued that there is resting going on in these later minutes. If the athlete has to complete the exercise at least once each minute then counting would stop after about 18 minutes with a total of about 245 completions.

7. (a) 1·2 m (b) Approx 9 cm (c) 8 m **8.** 11·67 m

Miscellaneous Exercise Four. Page 85.

1. (a) 2^6 (b) 2^8 (c) 2^7 (d) 2^2 (e) 2^{10} (f) 2^2 (g) 2^{14} (h) 2^0 (i) 2^5

2. $\frac{1}{2}$ **3.** 16 **4.** $\frac{9}{4}$ (i.e. 2·25)

5. 1 **6.** 64 **7.** $\frac{1}{25}$ (i.e. 0·04)

8. $\frac{1}{3}$ **9.** 5 **10.** $\frac{1}{7}$

11. Compare your reasoning with that of others in your class.

12.

	$T_1, T_2, T_3, T_4, T_5, \ldots$	Recursively defined.
(a)	17, 22, 27, 32, 37, ...	$T_n = T_{n-1} + 5,\ T_1 = 17$
(b)	100, 93, 86, 79, 72, ...	$T_{n+1} = T_n - 7,\ T_1 = 100$
(c)	5, 15, 45, 135, 405, ...	$T_n = 3T_{n-1},\ T_1 = 5$
(d)	6, 10, 14, 18, 22, ...	$T_{n+1} = T_n + 4,\ T_1 = 6$
(e)	2, 6, 18, 54, 162, ...	$T_{n+1} = 3T_n,\ T_1 = 2$
(f)	17, 9, 1, −7, −15, ...	$T_{n+1} = T_n - 8,\ T_1 = 17$

13. $a = 3, k = 2$, for (a) $T_{20} = 524\,288$, for (b) $T_{20} = 62$

14. (a) 15

15. (a) $x = 29,\ T_{n+1} = T_n + 21,\ T_1 = 8$
(b) $x = 20,\ T_{n+1} = 2{\cdot}5 \times T_n,\ T_1 = 8$. Or: $x = -20,\ T_{n+1} = -2{\cdot}5 \times T_n,\ T_1 = 8$.

16. (a) When $t = 3{\cdot}493$ (to three decimal places), i.e. in approximately 3·5 years.
(b) When $t = 6{\cdot}986$ (to three decimal places), i.e. in approximately 7 years.

17. $T_1 = 30,\ T_{n+1} = T_n + 3$.
One day prior to the championships Rosalyn will practise for 90 minutes.
During the 20 days prior to the championships Rosalyn will practise for a total of 20 hours and 30 minutes.

18. After 20 years account A will have a balance of \$3207135 (nearest dollar) compared to account B which after 20 years will have a balance of \$1949636 (nearest dollar).
The organisers need to have \$607906 available "now". (Rounded up to next dollar.)

Exercise 5A. Page 90.

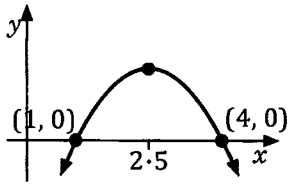
1. (a) A→B, D→F (b) B→D, F→H, H→I (c) B,D,F,H

2. I: B, C, D, E, F, G II: A III: H IV: A, D, G
V: A, D, H VI: C, F VII: A, E, G VIII: A, B, F, G
IX: A, E, F, G X: B, F, H

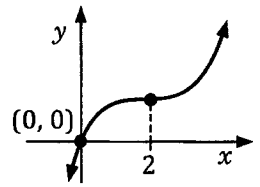
3. (a) C, E, H, K, M, O. (b) A, B, I, J, N, P. (c) D, F, G, L.

4. (a) 2 (b) 4 (c) 0 (d) −2
(e) −4 (h) 2 (g) 2

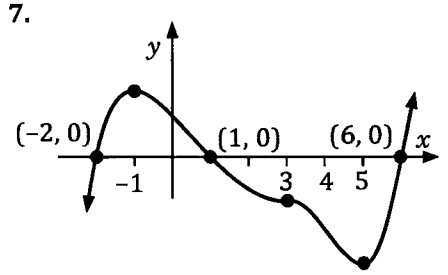
5.



6.



7.



Exercise 5B. Page 94.

1.

Point P	Point Q	Grad of chord PQ
(2, 4)	(4, 16)	6
(2, 4)	(3, 9)	5
(2, 4)	(2·5, 6·25)	4·5
(2, 4)	(2·1, 4·41)	4·1
(2, 4)	(2·01, 4·0401)	4·01
(2, 4)	(2·001, 4·004001)	4·001
(2, 4)	(2·0001, 4·00040001)	4·0001

Thus the gradient of $y = x^2$ at $x = 2$ is 4.

2. $y = x^2$

x	0	1	2	3	4	5
grad	0	2	4	6	8	10

Compare you rule with those of others.

3. $y = 3x^2$

x	0	1	2	3	4	5
grad	0	6	12	18	24	30

Compare you rule with those of others.

Exercise 5D. Page 103.

1. $2x$ **2.** $3x^2$ **3.** 1 **4.** $4x^3$
5. 0 **6.** $12x$ **7.** $24x^3$ **8.** 7
9. 16 **10.** $14x^6$ **11.** $14x$ **12.** 9
13. $\frac{x}{5}$ **14.** $4x^5$ **15.** $9x^5$ **16.** $2x^6$
17. $8x$ **18.** $20x^3$ **19.** $24x^2$ **20.** 0
21. $7x^6$ **22.** $24x^5$ **23.** $18x$ **24.** 5
25. 0 **26.** $18x^2$ **27.** $32x^3$ **28.** $15x^4$
29. $6x^5$ **30.** $42x^6$ **31.** $16x^3$ **32.** 10
33. 12 **34.** 12 **35.** 12 **36.** 80
37. 7 **38.** −20 **39.** 2 **40.** 0·8
41. (1, 1) **42.** (1, 1), (−1, −1) **43.** (1·5, 6·75) **44.** (0·5, 0·25), (−0·5, −0·25)
45. (1, 1) **46.** (−1, 1) **47.** $y = 6x - 4$ **48.** $y = -6x - 3$
49. $y = 20x - 20$ **50.** $y = -20x - 20$ **51.** $y = 16x - 24$ **52.** $y = 18x - 72$
53. (a) 24 (b) −3 (c) $9x^2$ (d) 36 **54.** (a) 6 (b) 24 (c) $3x$ (d) 6

55. (a) y changes by 234 units (from 16 to 250) when x changes from $x = 2$ to $x = 5$.
(b) y changes at an average rate of 78 units per unit change in x when x changes from 2 to 5.
(c) When $x = 2$ the instantaneous change in y is 24 units per unit change in x.
(d) When $x = 5$ the instantaneous change in y is 150 units per unit change in x.
56. (−1, 8) gradient −16, (2, 32) gradient 32.
57. (−2, −8) gradient 12, (0, 0) gradient 0, (2,8) gradient 12.
58. $\frac{1}{54}$, 1·5 **59.** $-\frac{1}{6}, \frac{1}{6}$.

Exercise 5E. Page 106.

1. $2x + 3$	**2.** $3x^2 - 4$	**3.** $12x - 21x^2$	**4.** $12x^3 + 6x^2 - 5$
5. $7 + 2x$	**6.** $12x - 3$	**7.** $8x + 7$	**8.** $15x^2 - 8x$
9. $20x^3 - 3$	**10.** $4x + 7$	**11.** $-6x + 7$	**12.** $1 + 2x + 3x^2 + 4x^3$
13. $-4 + 6x - 6x^2 + 4x^3$	**14.** −3	**15.** 24	**16.** 21
17. 11	**18.** $y = 7x - 4$	**19.** $y = 13x - 50$	**20.** $y = 8x - 34$
21. $y = 1$	**22.** (−5, 76), (1, −2)	**23.** (−3, 0) gradient −8, (5, 0) gradient 8.	
24. (5, − 10)	**25.** (−3, 20), (1, −4)		

Exercise 5F. Page 109.

1. $\frac{1}{2\sqrt{x}}$	**2.** $-\frac{1}{x^2}$	**3.** $-\frac{3}{x^2}$	**4.** $\frac{3}{\sqrt{x}}$
5. $\frac{2}{x^{\frac{2}{3}}}$	**6.** $\frac{3}{2}\sqrt{x}$	**7.** $\frac{2}{3x^{\frac{2}{3}}}$	**8.** $-\frac{3}{x^4}$
9. $-\frac{4}{x^5}$	**10.** $-\frac{6}{x^4}$	**11.** $-\frac{20}{x^5}$	**12.** $2x + \frac{1}{2\sqrt{x}}$
13. $6x - \frac{2}{\sqrt{x}}$	**14.** $1 - \frac{1}{x^2}$	**15.** $2x + \frac{2}{x^3}$	**16.** $\frac{1}{2\sqrt{x}} - \frac{3}{x^2}$
17. $2x + 1 - \frac{1}{x^2} - \frac{2}{x^3}$	**18.** $-\frac{2}{x^2}$	**19.** $-\frac{3}{2\sqrt{x^3}}$	**20.** $-\frac{2}{x^{\frac{4}{3}}}$
21. $-\frac{1}{3x^{\frac{4}{3}}}$	**22.** −5	**23.** 0·25	**24.** 0·0625
25. 11	**26.** $\frac{8}{3}$	**27.** −4	**28.** −0·5
29. (2, 0·5), (−2, −0·5)	**30.** (0·25, 0·5)	**31.** (9, −243)	**32.** $y = 0{\cdot}25x + 1$
33. $y = -x + 2$	**34.** $y = -0{\cdot}25x + 0{\cdot}75$	**35.** $\left(-1\frac{1}{3}, -1\frac{11}{12}\right), \left(1\frac{1}{3}, 1\frac{11}{12}\right)$	

36. Answers not given here. Discuss with others in your class and your teacher.

Miscellaneous Exercise Five. Page 111.

1. (a) 4 (b) 16 (c) 3 (d) 7 (e) 9 (f) 4
(g) 4 (h) 7 (i) 3 (j) 3 (k) 0 (l) 4
(m) 2 (n) 4 (o) 9
2. (a) 9 (b) 16
3. (a) 13 (b) −24
4. (a) 1024 (b) 1073741824 (c) 2046 (d) 2147483646
5. (a) $-3x^2$ (b) $10x - \frac{3}{\sqrt{x}}$ (c) $10x - \frac{2}{x^3}$ **6.** −4

7. (a) Reciprocal. $y = \frac{-6}{x}$ (b) Quadratic. $y = x^2 + 1$

(c) Linear. $y = 3x + 5$ (d) Exponential. $y = 5^x$

(e) Quadratic. $y = x(x + 1)$ (f) Exponential. $y = 10^x$

(g) Exponential. $y = 4 \times 2^x$, i.e. $y = 2^{x+2}$ (h) Reciprocal. $y = \frac{-24}{x}$

(i) Cubic. $y = 2x(x + 3)(x - 3)$

8. The other two angles of the triangle are of size 60° and 110°.

9. $T_1 = 0{\cdot}8$, $T_{n+1} = 5 \times T_n$.

10. (a) 8×10^{11} (b) 8×10^{11} (c) 8×10^{21}

(d) $1{\cdot}6 \times 10^9$ (e) 2×10^{-3} (f) 5×10^2

11. (a) Sequence 1: 5, 17, 53, 161, 485.
Sequence 2: 0·125, 0·25, 0·5, 1, 2.
Sequence 3: −5, 5, 15, 25, 35.

(b) Sequence 1: Neither. Sequence 2: Geometric. Sequence 3: Arithmetic.

(c) Sequence 1: 721 Sequence 2: 3·875 Sequence 3: 75.

(d) Sequence 1: 774 840 977 Sequence 2: 16 384 Sequence 3: 165.

(e) Sequence 1: 1 162 261 446 Sequence 2: 32 767·875 Sequence 3: 1440.

12. (a) $y = 5x - 1$ (b) $y = 23x - 29$ is tangent at (2, 17) and $y = 23x + 35$ is tangent at (−2, −11).

13. (a) The display tells us that from $x = 1$ to $x = 6$ the function has an average rate of change of 64.

(b) The display tells us that at $x = 5$ the function has an instantaneous rate of change of 105.
I.e., at $x = 5$, $\frac{df}{dx} = 105$.

14. C **15.** (a) 3 (b) positive (c) negative

16. Required x-coordinates are 4 and 6. Required y-coordinate is 2·816

Exercise 6A. Page 117.

1. $10r + 3$ **2.** $3 + 6k - 18k^2$

3. $15r^2 - 2r + 15$ **4.** $8p^3 + 9p^2 - 14$

5. $36t^2 + 18t - 8$ **6.** (a) 16 (b) 26 (c) 36

7. (a) 12 (b) 18 (c) −24 **8.** (a) 20π (b) 6π (c) 140

9. (a) 32π (b) 48π (c) 60π **10.** (a) 4π (b) 36π (c) 400π

11. (a) $\frac{4\pi t^2}{25}$ m^2 (b) 0·64π m^2 (c) $\frac{8\pi t}{25}$ m^2/s (d) 0·96π m^2/s

12. (a) 120 (b) 3870 (c) 750 bct/h (d) $(500 + 30t^2)$ bct/h
(e) (i) 620 bct/h (ii) 1250 bct/h (iii) 3500 bct/h

13. (a) 400 (b) 50 units/h (c) 8
(d) (i) 57 units/h (ii) 66 units/h (iii) 69 units/h

14. (a) (i) 0·2 L (ii) 2088 L (b) (i) 0·03 L/min (ii) 0·25 L/min (iii) 2·89 L/min

15. (a) (i) 42 (ii) 44 (iii) 47 (iv) 70 (b) $(0{\cdot}2t + 2)$ deer/yr
(c) (i) 3 deer/yr (ii) 4 deer/yr (iii) 6 deer/yr

16. (a) 150 000 tonnes (b) 48 000 tonnes

(c) Rate of decrease $= 8000 + 840t - 60t^2$ i.e. $\frac{dT}{dt} = (60t^2 - 840t - 8000)$ tonnes/year

(d) (i) 9440 tonnes/year (ii) 10 400 tonnes/year (iii) 10 940 tonnes/year

17. (a) 1000 cm^3 (b) 992·4 cm^3 (c) $(0{\cdot}2t - 4)$ cm^3/s (d) (i) −4 cm^3/s (ii) −3·4 cm^3/s
(e) 20 seconds (f) $a = 0$, $b = 20$.

Exercise 6B. Page 126.

(Sketches not given here - check using a graphic calculator).

1. (c) (−3, 61)

2. (c) (3, −50·5)

3. Local maximum point at (−3, 20). Local minimum point at (1, −12).

4. Local maximum point at (1, 37). Local minimum point at (5, 5).

5. Local (and global) maximum point at (2, 9).

6. Horizontal inflection at (0, 0).

7. Local (and global) minimum point at (0, 0).

8. Local minimum point at (0, 0). Local maximum point at (2, 4)

9. Local (and global) minimum point at (1, 5).

10. Local (and global) min at (−2, −22). Local max at (0, 10). Local min at (1, 5).

11. (a) (0, 0)

(b) (−3, 0), (0, 0)

(c) As $x \to +\infty$, $y \to +\infty$. As $x \to -\infty$, $y \to -\infty$.

(d) Local maximum point at (−3, 0). Local minimum point at (−1, −4).

(e) Use a graphic calculator to check validity of your sketch.

(f) Minimum value is −20. Maximum value is 16.

12. Local maximum point at (0, 0). Local minimum point at (1, −1). (a) −1 (b) −5

Exercise 6C. Page 130.

Each solution should clearly show the use of calculus and justify a maximum (or minimum) value.

1. When $t = 8$, X has a local minimum value of 16.

2. When $p = 10$, A has a local maximum value of 300.

3. The maximum value of A is 20 and it occurs when $x = 10$ and $y = 2$.

4. The maximum value of A is 13·5 and it occurs when $x = 4{\cdot}5$ and $y = 3$.

5. When $x = 35$ the maximum profit of \$725 is realised.

6. When $x = 120$ the maximum profit of \$9400 is realised.

7. (a) 25 m × 25 m (b) 50 m × 25 m (with the existing wall forming one of the 50 m sides).

8. The manufacturer should spend \$30 000 on advertising to achieve the max profit of \$140 000.

9. For maximium capacity the dimensions need to be width 0·4m, length 0·6 m, height 0·5 m.

10. The maximum capacity is achieved when $x = 10$.

11. 6 cm should be turned up along each edge to maximise the capacity of the gutter.

12. (a) $\$(2500 + 500x - 25x^2)$ (b) 10 (c) \$2 (d) 5000 (e) \$5000

13. Minimum N is 1100, to the nearest 100. (When $t = 16$).
Maximum N is 4600, to the nearest 100. (When $t = 24$).

14. (a) The body is 105m from the origin after three seconds.

(b) $(t^2 - 12t + 50)$ m/s

(c) The initial velocity of the body is 50 m/s.

(d) When $t = 6$ the body is moving with minimum velocity and the body is then 156 metres from the origin.

15. 81 mm, 880 cm

16. The owner should spend \$12500 on security.

Exercise 6D. Page 134.

1. $6r - \frac{5}{r^2}$.

2. (a) 100 (b) 40 (c) 20

3. Maximum at $(-\sqrt{2}, -2\sqrt{2})$, minimum at $(\sqrt{2}, 2\sqrt{2})$.

4. Minimum at (−2, 9), maximum at (2, 1).

5. Maximum at (−4, −18).

6. (a) $\frac{500}{x^2}$ (b) $\left(x^2 + \frac{2000}{x}\right)\text{cm}^2$ (c) $10, 5, 300\,\text{cm}^2$

7. Correct to one decimal place, the base radius needs to be 4·4 cm and the height 8·8 cm.

8. When the base radius is 3·5 cm (correct to one decimal place) and the height is 14·0 cm (correct to one decimal place) the cost of material is minimised.

Miscellaneous Exercise Six. Page 135.

1. (a) 5^2 (b) 5^4 (c) 5^3 (d) 5^0 (e) 5^3 (f) 5^6
(g) 5^5 (h) 5^4 (i) 5^7 (j) 5^3 (k) 5^2 (l) 5^1
(m) 5^{10} (n) 5^4 (o) 5^{17} (p) 5^2 (q) 5^6 (r) 5^3
(s) 5^5 (t) 5^6 (u) 5^7 (v) 5^8 (w) 5^2 (x) 5^3
(y) 5^2 (z) 5^2

2. (a) a^4 (b) $\frac{5a^7}{b^8}$ (c) $2 + 2^n$ (d) $x + 2x^4$ (e) 2^x (f) $\frac{3}{5}$

3. The first six terms are 97, 108, 119, 130, 141, 152.

4. The first five terms are 350, 70, 14, 2·8, 0·56.

5. (a) 6 (b) 3 (c) 9

6. (a) 60 (b) 24 (c) 105

7. 15 (a) 55 (b) 120

8. (a) 14 535 (b) −442 860 (c) 12 582 906 (d) 500 (e) 5

9. From the graph, some of the points the tangent at $x = 1$ seems to pass through are (−3, −11), (0, −2), (1, 1) and (3, 7). Thus the tangent at $x = 1$ has a gradient of 3. Thus gradient of f(x) at $x = 1$ is 3.

From the graph, some of the points the tangent at $x = 2$ seems to pass through are (0, −16), (2, 8) and (3, 20). Thus the tangent at $x = 2$ has a gradient of 12. Thus gradient of f(x) at $x = 1$ is 12.

Finding the gradient of $y = x^3$ at $x = 1$ and at $x = 2$ using calculus confirm these values.

10. 2

11. (a) 10 (b) 3 (c) 13

12. (a) D, H, K, P (b) B, F, I, K, N, O (c) G, H, L, M (d) A, C, D, E, J, P

13. At the point (1, 2).

14. (a) (−1, −3), (1, 3) (b) (0·25, −1·25)

15. (a) 343 062, 1 698 992, 5 308 522 (b) 67 513, 223 973, 526 233

16. $y = 25x + 185$ (at the point (−5, 60)) and $y = 25x - 71$ (at the point (3, 4)).

17. (a) $a = -20$, $c = 260$ (b) $\$(260p - 20p^2)$
(c) $\$(400p - 20p^2 - 1820)$ (d) 10, 60, \$180

18. (a) $1{\cdot}4 \times 10^{24}$ (b) $7{\cdot}2 \times 10^{12}$ (c) $6{\cdot}8 \times 10^{12}$
(d) $3{\cdot}5 \times 10^{1}$ (e) 7×10^{24} (f) $2{\cdot}45 \times 10^{14}$

19. (a) 200 (b 1500 (c) 130
(d) (i) 30 organisms/h (ii) 105 organisms/h (iii) 330 organisms/h

20. (a) The display tells us that from $x = 1$ to $x = 3$ the given function has an average rate of change of 41.

(b) The display tells us that at $x = 3$ the given function has an instantaneous rate of change of 109. I.e., at $x = 3$, $\frac{df}{dx} = 109$.

21. (a) $\$(10 - 0{\cdot}2x)$ per metre (b) $(500 + 25x)$ metres
(c) $\$(5000 + 150x - 5x^2)$
(d) 15. Negative coefficient of x^2 in the quadratic, hence turning point is a maximum.

Exercise 7A. Page 142.

1. $\frac{1}{8}x^8 + c$ **2.** $\frac{1}{6}x^6 + c$ **3.** $\frac{1}{5}x^5 + c$ **4.** $\frac{1}{4}x^4 + c$ **5.** $\frac{1}{3}x^3 + c$

6. $\frac{1}{2}x^2 + c$ **7.** $x + c$ **8.** $4x^3 + c$ **9.** $2x^6 + c$ **10.** $2x^4 + c$

11. $7x^2 + c$ **12.** $6x + c$ **13.** $x^3 + 3x^2 + c$ **14.** $2x^3 - x + c$ **15.** $7x + 3x^4 + c$

16. $3x^2 - 3x^5 + c$ **17.** $7x - 4x^2 + c$ **18.** $\frac{x^3}{3} + 3x + c$

19. $3x^6 + x + c$ **20.** $2x^3 + \frac{1}{2}x^2 + c$ **21.** $4x^3 + 2x^4 + 2x + c$

22. $x^3 - x^2 + \frac{1}{6}x^6 + c$ **23.** $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + c$ **24.** $3x^4 + 3x^2 + 5x + c$

25. $x^3 + 5x^2 + 8x + c$ **26.** $3x^3 + 4x^2 - x + c$ **27.** $\frac{1}{3}x^3 - 4x + c$

28. $\frac{1}{3}x^3 - x^2 - 3x + c$ **29.** $2x^4 + x^3 + c$ **30.** $x^4 + 4x^3 + 2x^2 + c$

31. $y = 2x^3 + 7$ **32.** $y = \frac{3}{2}x^2 + 2x - 2$ **33.** $y = x^3 - x^2 + 6$

34. $y = 2x^3 - 5x + 3$ **35.** $y = 3x + 2x^4 + 7$ **36.** (a) $f(x) = \frac{1}{2}x^3 + 2x^2 - x - 2$ (b) 8

37. (a) $\frac{3}{2}x^2 - 6x + 6$ (b) 24 (c) $a = -4$ or 8 **38.** p = 27

39. (−4, 0), (0, 0), (4, 0) **40.** k = −1

Exercise 7B. Page 144.

1. $V = 3t^2 + 5t + 30$ **2.** (a) $x = t^2 - 6t + 7$ (b) 23 (c) 1 or 5

3. (a) $A = 2r^2 + 3r^4 + 2$ (b) 58 **4.** (a) $C = x^2 + 3x + 100$ (b) $C = x^3 + x^2 + 5000$

5. (a) $R = 50x$ (b) $R = 50x - 0{\cdot}025x^2$ **6.** \$38000

7. $(7000 - 20t - 5t^2)\ \text{cm}^3$ **8.** Increasing. $A = 100t + 10000$

9. $C = 40x + 1000$ **10.** $R = 200x - \frac{1}{20}x^2$, 150000

11. (a) 29 (b) 43 (c) 176

Exercise 7C. Page 148.

1. $\frac{x^3}{3} + c$ **2.** $\frac{x^2}{2} + c$ **3.** $\frac{x^4}{4} + c$ **4.** $2x + c$

5. $2x^5 + c$ **6.** $2x^4 + c$ **7.** $2x^2 + x + c$ **8.** $2x^3 - 5x + c$

9. $4x^2 - 7x + c$ **10.** $\frac{x^2}{2} + 3x^3 + c$ **11.** $\frac{x^2}{2} - x + c$ **12.** $2x^3 + \frac{11x^2}{2} + 3x + c$

13. $2x^3 + 3x^2 + c$ **14.** $2x^4 - x^3 + c$ **15.** $\frac{3x^4}{2} + 4x^3 + 3x^2 + c$

Miscellaneous Exercise Seven. Page 148.

1. 10^4 **2.** 10^{-1} **3.** 10^6
4. 10^8 **5.** 10^2 **6.** 10^6
7. $10^{0.5}$ **8.** $10^{1/3}$ **9.** $10^{1.5}$
10. 0 **11.** 5 **12.** 5
13. 4 **14.** 5 **15.** 4
16. 5 **17.** 8 **18.** 2

19. $T_1 = 10$, $T_{n+1} = T_n + 6$.
The sum of the first fifteen terms exceeds the fifteenth term by 686.
(I.e. by the sum of the first 14 terms.)

20. (a) 0 (b) 5 (c) $10x + 5$ (d) $15x^2 + 10x + 5$ (e) $2x + \frac{1}{2\sqrt{x}}$ (f) $-\frac{3}{2x^4}$

21. (a) 29 (b) 9 (c) $8x - 3$ (d) 21

22. 8 **23.** 6 **24.** (−2, 3000), (16, 84)

25. (a) $y = 5^x + 1$ is $y = 5^x$ with 1 added to the right hand side.
Thus the graph of $y = 5^x + 1$ that of $y = 5^x$ translated up one unit.

(b) $y = 5^{x+1}$ is $y = 5^x$ with the x replaced by $x + 1$.
Thus the graph of $y = 5^{x+1}$ is that of $y = 5^x$ moved left 1 unit.
Alternatively we could write $y = 5^{x+1}$ as $y = 5 \times 5^x$ which is $y = 5^x$ with the right hand side multiplied by 5. Thus the graph of $y = 5^{x+1}$ is also that of $y = 5^x$ dilated parallel to the y-axis, scale factor 5.

(c) $y = 5^{-x}$ is $y = 5^x$ with the x replaced by $-x$.
Thus the graph of $y = 5^{-x}$ is that of $y = 5^x$ reflected in the y-axis.

(d) Writing $y = \frac{1}{5^x}$ as $y = 5^{-x}$ we see that the answers to this part will be as for part (c).

26. 9

27. (a) (−1, −2) (b) (−1, −3), (1, 3)

28. (a) a = 7, b = 3 (b) (0, −21) (c) Gradient is −10 at (−3, 0). Gradient is 10 at (7, 0)
(d) (5, −16) (e) $y = -4x - 21$

29. a = 3, b = 4. Gradient at P is −7. Gradient at Q is 7. Gradient at R is 1.

30. (a) $(2000 - 40x + 0{\cdot}2x^2)$ dollars per unit (b) \$500 per unit
(c) $P(x) = 500x - 20000 + 20x^2 - \frac{x^3}{15}$ dollars (d) \$2000 per unit

31. From the display we can conclude that the graph of $y = 4x^3 + 9x^2 - 210x + 75$ has two stationary points, one is at (−5, 850) and the other is at (3·5, −378·25).

32. (a)

Width(cm)	Length(cm)	Height(cm)	Volume(cm^3)
10	20	120	24000
20	40	90	72000
30	60	60	108000
40	80	30	96000

A continuation of the table, for suitably chosen values for the width, leads to maximum volume achieved when, to the nearest cm, the width is 33 cm, the length is 66 cm and the height is 51 cm.

(b) Volume $= 300w^2 - 6w^3$.
Calculus, and consideration of the graph of $f(w) = 300w^2 - 6w^3$, confirms that volume is maximised for $w = \frac{100}{3}$ cm, i.e. 33 cm (nearest cm).

33. Base 4 m, height 8 m, area $32m^2$.

Exercise 8A Page 155.

1. (Graph not shown here.)
(a) The car reaches C at 11.54 a.m. and truck reaches town C at 12.15 p.m.
(b) From 8.30 a.m. to 9.30 a.m. the truck maintained a steady speed of 100 km/h.
(c) The average speed of the truck from A to B was 87 km/h (to the nearest km/h).
(d) The car passes the truck at 10.30 a.m. in town B, just as the truck is about to leave B.

2.

A	2m	2 m	6 m/s	6 m/s
B	6m	6 m	0 m/s	0 m/s
C	8m	8 m	2 m/s	-2 m/s
D	5m	-5 m	5 m/s	5 m/s
E	9m	-9 m	3 m/s	-3 m/s
F	1m	-1 m	7 m/s	-7 m/s

3. (a) 4 m/s (b) 31 m/s
4. (a) 6 m (b) 5 m/s (c) 23 m/s
5. (a) 0 m (b) -3 m/s (c) 3 m/s
6. (a) 0 m (b) 1 m/s (c) 37 m/s
7. (a) 3 m (b) 6 m/s (c) 6 m/s
8. (a) -3 m (b) -20 m/s (c) 8 m/s
9. (a) 1 m (b) -6 m/s (c) 150 m/s
10. 17 m, 14 m/s
11. 2 m, 4 m/s
12. (a) 8 m (b) 8 m/s (c) 7
13. (a) 6 m (b) -1 m/s (c) 4
14. (a) 12 m (b) 8 m/s (c) 3
15. 153 m
16. (a) 45 m (b) 105 m (c) 10 (d) 60 m (e) 15 m
17. (a) 8 m (b) 20 m (c) 4 (d) 20 m (e) 3 m
18. (a) 40 m (b) 52 m (c) 6 (d) 20 m (e) 7 m
19. (a) 94 m (b) 148 m (c) 5 (d) 162 m (e) 26 m
20. (a) 10, 0 (b) 120 m, 10 m/s upwards (c) 0 m/s, 125 m
21. 60 m/s
22. (a) 12 (b) 318 m (c) A and B collide "head-on".

Exercise 8B Page 159.

1. 23 metres
2. 34 metres
3. The body is at the origin when $t = 0{\cdot}6$ and when $t = 4$.
4. -5 metres
5. At the origin the velocity of the body is -6m/s, when $t = 2$, and 6 m/s, when $t = 4$.
6. (a) When $t = 5$.
(b) -12·5 metres
7. 19 metres
8. 0·5 metres

Miscellaneous Exercise Eight. Page 159.

1. (a) (0, -6) (b) (0, 15)
(c) (0, 6) (d) (0, 3)
(e) (0, -4) (f) (0, 12)
2. (a) (3, 0) (b (-3, 0)
(c) (-3, 0), (3, 0) (d) (2, 0), (7, 0)
(e) (3, 0), (-2, 0), (3·5, 0) (f) (1, 0), (-1, 0), (3, 0), (-4, 0)

3. 2^{15} **4.** 2^{-4} **5.** 2^{-6} **6.** 2^{-1}

7. 2^{-2} **8.** 2^{10} **9.** 2^{13} **10.** 2^{6}

11. 2^{6} **12.** 2^{3} **13.** 2^{0} **14.** 2^{8}

15. $n = -5$ **16.** $n = 5$ **17.** $n = 13$ **18.** $n = 6$

19. $n = 10$ **20.** $n = \frac{5}{6}$ **21.** $n = 3$ **22.** $n = 1$

23. $n = 2$ **24.** 15, 21, 27, 33, 39.

25. 100, 93, 86, 79, 72. **26.** 4, 20, 100, 500, 2500.

27. 6, 24, 96, 384, 1536. **28.** 213, 219, 225, 231, 237.

29. 256, 384, 576, 864, 1296.

31. (a) $6x$ (b) $15x^2$ (c) $x + 3$ (d) $30x - 1$ (e) $6 + 18x$ (f) $-2x$

32. (a) $14x$ (b) $6x^2$ (c) $12x^3 + 3x^2 - 10x + 9$ (d) $9x^2 - 4x + 3$

33. 7

34. (a) 10 (b) 6 (c) $6x + 2$ (d) 14

35. (a) (0·5, 1·25) (b) (−2, −40), (2, 40) (c) (2, 10) (d) (−2, −20), (4, 16)

36. (a) −3 (b) −0·375 **37.** At the point (1, −4). **38.** $y = -2x - 1$

39. (a) From $t = 0$ to $t = 5$ the object travels 3 metres. The average speed is 0·6 m/sec.
(b) From $t = 5$ to $t = 8$ the object travels 12 metres. The average speed is 4 m/sec.
(c) When t = 5 the speed of the object is 1 m/sec.
(d) When t = 8 the speed of the object is 3 m/sec.

40. (a) −12 m/s (b) 12 m/s (c) 2·5 (d) 1·5, 2·5

41. a = 3, b = 5. (−1, −4) **42.** $y = 1{\cdot}25x - 2$

43. $f(x) = 7x^3 - 15x^2$, $f(3) = 54$, $f'(2) = 24$

44. (a) 3399, 9744, 235056 (b) 859, 1729, 14401

45. (a) $60x + c$ (b) $30x^2 + c$ (c) $20x^3 + c$ (d) $15x^4 + c$
(e) $12x^5 + c$ (f) $10x^6 + c$ (g) $2x^4 - 5x^3 + 2x + c$
(h) $4x - \frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 + c$ (i) $\frac{1}{3}x^3 - 9x + c$ (j) $12x^4 - 8x^3 + c$

46. (a) $y = 2x^2 - 3x + 3$ (b) $y = 2x^3 - x^2 + 4x + 7$
(c) $y = 2x^4 - 4x^3 - 2x^2 + 11x - 10$

47. When $x = 2$, $y = 64$.

48. (a) 7500 cm^3 (b) 29100 cm^3
(c) $(5400 - 900t + \frac{75t^2}{2})$ cm^3/s (d) 3750 cm^3/s, 2400 cm^3/s, 150 cm^3/s.

49. (a) y changes by 320 units (from 1 to 321) when x changes from $x = 0$ to $x = 10$.
(b) y changes at an average rate of 32 units per unit change in x when x changes from 0 to 10.
(c) When $x = 0$ the instantaneous change in y is 2 units per unit change in x.
(d) When $x = 10$ the instantaneous change in y is 62 units per unit change in x.

50. D **51.** \$18, 15000

52. (a) 5 metres (b) 57 metres
(c) From 5 metres to 57 metres in 4 seconds is an average rate of change of 13 m/sec.
(d) When $t = 2$ the instantaneous rate of change of x is 3 m/sec.
(e) When $t = 6$ the instantaneous rate of change of x is 27 m/sec.

53. (a) $250 - \pi r$ (b) $A = 500r - \pi r^2$ **54.** \$22003·73

55. (a) $120 - x$ (b) $\$(360 + 3x - 0{\cdot}05x^2)$ (c) \$4·50 (d) \$405
(e) $P(x) = \$(120 + 5x - 0{\cdot}05x^2)$ (f) \$5·50 (g) \$245

56. (a) 25 m (b) 5 (c) 20 m

57. $a = -0.02$, $b = 4{\cdot}8$, $c = 0{\cdot}018$, $d = -6{\cdot}6$, $e = 50$, $f = 30$, $g = 20$, $h = \frac{550}{3}$